

Erodibility in Centrifuge Models of Embankment Overtopping

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The mechanisms that take place at the instant of dam overtopping, leading to an eventual breaching and failure, depend on many factors—breach dimension, time, soil conditions, reservoir size and inflow, and overflow depth are some of the many variables which contribute to the severity of overtopping failures in embankments. Centrifuge tests can be used effectively for parameter studies in which material properties and behavior depend on variations in boundary loads and conditions. To examine embankment overtopping a series of six centrifuge tests were conducted and compared to large-scale, 1G testing conducted by [Hanson et al. \(2005\)](#). An examination of the scaling laws associated with time was performed comparing the width of headcut at the crest of the embankment between the two sets of data. After establishing a time-scaling factor, an analysis of cumulative volume loss and erodibility was conducted.

Notation

d_c	Critical depth (m)
g	Acceleration due to gravity (m s^{-2})
h	Normal depth of flow on slope (m)
k_d	Detachment rate coefficient ($\text{m}^3 \text{N}^{-1} \text{hr}^{-1}$)
n	Manning coefficient ($\text{s m}^{-1/3}$)
S	Energy grade line slope (unitless)
t	Time (hr)
W	Width of headcut (m)
γ	Unit weight of water (N m^{-3})
τ_c	Critical shear stress (Pa)
τ_e	Hydraulically applied boundary stress (Pa)

1. Introduction

Earthen embankments such as dams, levees, lagoons, and dikes are naturally occurring or man-made structures found around the world along lakes, reservoirs, rivers, and coastal waterways. The former are formed by sedimentary processes whereas artificial embankments are constructed primarily from naturally erodible materials such as zoned earth-fill, rock-fill, and clay (?). The US Army Corps of Engineers (USACE) alone for example, maintain more than 22,530 km of such earthen embankments across the United States ([USACE, 2018](#)). These structures retain water and subsequently have several important societal impacts including: flood and water supply management, sediment control, power generation, and navigation. Like all structures, earthen embankments are subject to aging and decay which increases their probability of failure under extreme hydrologic loading events –

posing significant downstream risk to human life, property damage, and service interruption (?).

Embankment failure commonly occurs according to one of three primary mechanisms – overtopping, internal erosion, or piping ([Foster et al., 2000](#)). The former has been attributed to more than 40% of earthen embankment failures in the last 70 years ([Linsley and Franzini, 1981](#); [Talukdar and Dey, 2019](#)). Overtopping occurs when inflow rates into a body of water surpasses the outflow rate (i.e., a flood event) causing the water level to rise and overtop the crest of an embankment. This leads to progressive surface erosion once the shear forces associated with the overtopping water exceeds the resistive forces of the soil comprising the embankment ([Hanson and Temple, 2002](#); ?). Mathematically, the process of overtopping is “an unsteady, nonhomogeneous, nonlinear, three-dimensional problem” which requires substantial understanding of soil-water interaction ([Chow, 1959](#)). It is dependent on a wide range of interrelated parameters including but not limited to: embankment size and geometry (e.g., slope), soil conditions (texture, cohesion, moisture content, compaction, etc.), presence of vegetation, as well as flood/overtopping depth and duration ([Talukdar and Dey, 2019](#)). Current understanding of overtopping is derived primarily from historic post-failure investigations and relatively small 1G experiments; capturing natural overtopping events is difficult if not impossible as they are often remote, unpredictable, and lack instrumentation needed to conduct a comprehensive qualitative analysis. The reader is referred to ? and [Flynn et al. \(2021\)](#) for a review of historical overtopping case studies and Table 1 in ?

for a compiled summary of laboratory embankment breach tests. While still informative with respect to soil conditions present at a site, post-failure investigations provide no detail to the processes and mechanisms occurring during a failure event. 1G laboratory experimentation can be similarly constrained; often logistically limited to a small number of tests and an inability to replicate the stress states associated with large embankment structures under gravity loading (?).

Centrifuge testing is well-suited for studying the mechanisms related to overtopping failure. The associated headcutting process (i.e., erosion) can be studied under the correct stress state and captured in real time using state-of-the-art sensing techniques in a repeatable, cost-effective manner (time and economic). The feasibility of using centrifuge modeling in the context of overtopping was first demonstrated by Ko et al. (1989) for two prototype embankments. More recent studies performed by Sun et al. (2012), ?, and Takahashi et al. (2019) have explored various aspects of the problem. Irrespective of the focus, these studies are corroborative in that they show that centrifuge acceleration is necessary to produce stress states similar to those of larger embankments, thereby allowing the headcut erosion process to be adequately captured as required for a scaled model. While no single research method (e.g., traditional laboratory experimentation, field observation, physical modeling, or computational modeling) can address all the complexities of a real-world problem like overtopping, centrifuge modeling is an approach worth consideration and further exploration.

This centrifuge study has two primary objectives: (1) the continued improvement of the current understanding of overtopping failure by specifically looking at the relation between discharge, breach width, and eroded mass loss as a function of time; (2) the development of an economical and repeatable testing approach for the USACE tasked with defining hazard to property and life downstream of the embankments within its portfolio. The latter can greatly accelerate the research and development lifecycle while reducing the prohibitive costs, extensive logistical challenges, and safety considerations associated with full-scale testing. The centrifuge model design applied herein, specifically the soil properties, was based on the well-documented 1G experiments of Hanson et al. (2005). This provides a strong foundation for comparing and understanding the centrifuge experimental results as well as an opportunity to expand an existing dataset. The data resulting from the experiments presented herein can also provide much-needed, comprehensive, high-fidelity datasets under appropriate stress states suitable for model development, calibration, and validation.

2. Materials and Methods

2.1. Background

The present experimental study builds upon more than ten years of overtopping research conducted at the USDA-ARS Hydraulic Unit in Stillwater, Oklahoma between 1993 and 2011. This

comprehensive body of work associated with this effort involved laboratory JET testing, hydraulic flume experimentation, large-scale physical modeling, and computational modeling. When viewed collectively, the results and approaches showed a strong scale coherence with respect to estimated soil erodibility. The rates of headcut migration (dX/dt) and erosion widening (dW/dt) were observed to vary by three orders of magnitude based on the properties of the soil material as well as the compaction characteristics (Hanson et al., 2001, 2005; Hanson and Hunt, 2007; Hanson et al., 2010). The large-scale USDA-ARS physical overtopping testing was also instrumental in verifying the four-stage overtopping erosion process framework proposed by ?, which has since become the basis for overtopping analysis and modeling (Hunt et al., 2005). The reader is referred to Figure 7 in Hanson et al. (2005) for photographs depicting the four stages of overtopping which can be described accordingly as:

- **Stage I:** The initiation of overtopping flow (development of rill erosion) that results in the formation of a large headcut on the landward side of the embankment crest.
- **Stage II:** The migration of the headcut from the landward side to the water side of the embankment crest. The headcut begins to widen.
- **Stage III:** The embankment crest begins to erode and lower.
- **Stage IV:** The breach continues to widen while discharge continues until either the reservoir is drained or flood waters recede.

The test design for the centrifuge models described below was based on the large-scale embankments created by Hanson et al. (2005). The USDA-ARS group completed seven 1G overtopping tests on embankments comprised of three different soils: two non-plastic SM soils and one CL lean clay. The height of the embankments ranged from 1.5 to 2.3 meters. Replicating the embankment geometry, flow rates, and soil conditions as closely as possible in centrifuge experiments was a key design consideration. This facilitated the exploration of centrifuge time scaling through direct comparison with the Hanson et al. (2005) experiments.

2.2. Model Design

Figure 1 shows a cross-sectional view of the model set-up. The embankment height in the centrifuge tests were raised to 5 m prototype (20 cm model scale) in comparison to the Hanson et al, Table 1 (Hanson et al., 2005) experiments which were 2.3 m (Embankment 1) and 1.5 m (Embankment 2) in height. Direct scaling from these prototype embankments to the model scale would have resulted in a embankment only 6 or 9 cm in height; this would create a landward embankment face as small as 13 cm which would prevent flow from fully establishing and also cause the headcut erosion channel to potentially propagate too quickly. Increasing the height of the embankment is not expected to have a significant impact on the results as headcut formation, widening, and migration occurs primarily near the crest of an

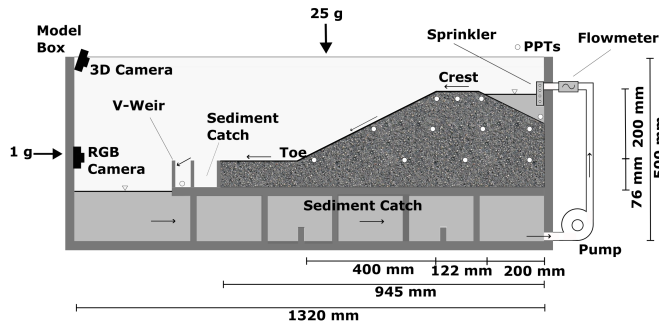


Figure 1. Layout in model scale.

embankment. Increasing the height of the embankment simply increases the amount of material that can erode from the primary headcut channel, thereby increasing the total volume loss. The embankment had a 2:1 slope in each experiment except SM Test 9 and CL Test 10 which had a 3:1 slope. Discharge rates similar to those reported in the experiments of [Hanson et al. \(2005\)](#) were used to ensure consistency between the datasets.

The ERDC's large beam centrifuge has a 6.5m radius with a maximum over- and under-stress error of only 1.19% for this model ([Madabhushi, 2017](#)). The effective radius of the centrifuge for this work was set to 2/3 of the embankment height to minimize errors in the under- and over-stress ratios. To avoid errors due to the radial gravity field, the slope of the levee is aligned with the center of the beam.

A self-priming impeller pump was used to control the water supplied to the model; a digital flow meter was used to monitor the inflow throughout the duration of the experiments. The water was stored in, and recirculated through, a storage chamber located beneath the raised platform on which the scaled embankment was constructed (Figure 1). A series of weirs were incorporated into the storage tank to remove sediment suspended in the water in order to prevent the pump from clogging. A large sediment catchment was located immediately downstream of the embankment to trap the majority of the debris eroding from the model, preventing it from interfering with the outflow through a v-notch weir. The outflow was determined by measuring water height in the v-weir chamber with a pressure transducer which was then used to calculate discharge through the v-notch weir ([USBR, 1997](#)).

The experiment design of this study was centered around the measurement of the headcut formation, migration, and widening over time; these data were in turn, used to determine total volume loss and erosion. An Intel RealSense D455 depth camera was mounted to the model container on the landward side of the levee (left side of Figure 1) to monitor the levee geometry during the overtopping event. The D455 employs stereoscopic techniques to create depth maps with a 2 mm accuracy. These depth maps,

Table 1. Soil properties

Soil	1	2
Unified Soil Classification	CL	SM
Grain size distribution ^a		
% sand (40/70) >0.75 mm	35	70
% silt (Vicksburg Silt) >0.002 mm	40	25
% clay (Kaolin) <0.002 mm	25	5
d ₁₀ (mm)	9x10 ⁻⁵	0.010
d ₅₀ (mm)	0.041	0.33
d ₆₀ (mm)	0.063	0.35
Atterberg Limits ^b		
Liquid limit	20.85	—
Plastic limit	12.7	—
Plasticity index	8.15	—
Compaction ^c		
Optimum water content (%)	10.2	7.3
Maximum dry density (g cm ⁻³)	1.93	2.02
Strength Characteristics ^d		
Friction angle (degrees)	23.2	42.7
Cohesion (kPa)	33.1	24.5

^a Measured according to ASTM standard D-422

^b Measured according to ASTM standard D-4318

^c Measured according to ASTM standard D-698

^d Measured according to ASTM standard D-3080

when represented as a three-dimensional point clouds, can be used to track and quantify surface deformation associated with the formation of the primary erosion channel and auxiliary rills and gullies. While the camera was used with great success in these experiments, it had several drawbacks. The camera was observed to be sensitive to changes in the model's brightness and contrast (e.g., water vs. soil) and produced large file sizes that could prove unwieldy to work with. In an effort to reduce file sizes, the camera was started and stopped several times throughout the experiments, which led to slight shifts in the reference frame that had to be addressed during post-processing.

2.3. Soil Properties

In the [Hanson et al. \(2005\)](#) tests, three soils were tested: two different non-plastic SM silty sand materials and a CL lean clay. Experiments conducted in the present study replicated the material properties of the CL soil and one of the SM soils. Key characteristics of the soils manufactured for the centrifuge experiments are presented in Table 1. Note that the properties for the CL soil had an identical grain size distribution and similar optimum water content (14%) and maximum dry density (1.78 g cm⁻³) as [Hanson et al. \(2005\)](#). The SM soil was designed to match Soil 1 in the USDA-ARS datasets; the reader is referred to Table 2 in [Hanson et al. \(2005\)](#) for soil property specifications.

Table 2. Centrifuge test matrix

Test Name	Soil Type	Density (g cm^{-3})	Pretest Water Content (%)	Inflow ($\text{m}^3 \text{s}^{-1}$)
Test 2	CL	1.78	9.3	1.26
Test 4	CL	1.72	11.6	1.22
Test 10*	CL	1.88	11.39	2.22
Test 5	SM	2.02	7.5	2.26
Test 7	SM	1.84	7.2	2.08
Test 9*	SM	2.26	7.2	1.57

*Test slope 3:1; all other tests conducted with a 2:1 slope

The model embankment was constructed in approximately 3 cm thick lifts. The soils were compacted near optimum moisture content (Table 1) using a modified 10.6 kg drop hammer. In an effort to concentrate the flow and erosion to the center of the levee and away from the boundaries of the box, the crest was curved slightly following the shape of the centrifuge's radial gravity field. Compaction tolerances were $\pm 6\%$ of maximum dry density for CL soils and $\pm 8\%$ for SM soils.

3. Results and Discussion

For this study, a series of six 25G centrifuge tests were conducted (see Table 2) at the Engineer Research and Development Center's (ERDC) Centrifuge Research Complex (CRC). This recently renovated 1,200 g-ton centrifuge has a 6.5m radius; the reader is referred to Bowman et al. (2022) for a full description of the machine and measurement capabilities. Note that the test conditions varied with respect to soil type, embankment slope (2:1 vs 3:1) as well as moisture content which fell on both the dry and wet side of the soils' optimum water content (OWC), an important consideration in embankment stability (Hunt et al., 2005).

3.1. Headcut Channel Formation

The temporal evolution of the headcut channel was tracked during each experiment using the D455 depth camera. Figure 2 presents an example cross-section across the width of the levee just downstream of the crest for SM Test 5 and CL Test 4. While the results from the other four tests are not visualized for brevity, the final headcut dimensions, water content, and total test duration are summarized in Figure 2 however. Note, the applied spatial and temporal scales corresponds to the model scale; the reader is referred to Section 3.2 for more details regarding scaling used in subsequent analysis.

The construction histories of many earthen embankments within portfolios managed by USACE and local municipalities are unknown, and their soil composition are uncertain. This is an important consideration, given that soil taxonomy plays a key role in determining an embankment's susceptibility to erosion and the evolution of a headcut during an overtopping event. This is emphasized when the final headcut dimensions and total test

duration of the six centrifuge tests are compared against one another (Figure 2). Headcutting began shortly after the water overtopped the embankment model, irrespective of soil type. A primary headcut channel formed immediately during the SM soil tests, whereas a series of rills developed in the CL tests and later coalesced into a larger channel. Over time, the headcut channel was observed to continuously grow and migrate up the landward face of the embankment. Note that positive spikes in the cross-sections (e.g., CL Test 4, 58.77 minutes) are indicative of the temporary accumulation of eroded material from further up the face. The headcut reached the crest of the embankment for each of the SM tests and CL Test 2 – indicative that overtopping was transitioning from Stage II to Stage III overtopping. The experiments had to be terminated shortly thereafter, or in the Case of CL Test 4 and CL Test 10, well before, because the sediment catchment basin (Figure 1) had filled; this impacted flow through the v-notch weir and led to fear that the suspended sediment in the water could damage the pump.

The overall progression of the channel (i.e., breach widening and scouring depth) occurred more slowly and to a lesser extent in the CL soils. Overall, these results indicate that embankments constructed from CL soil were typically more resistant to erosion than those constructed from SM soil, which is in good agreement with Hanson et al. (2005). The mean duration of the SM tests was approximately 40% that of the CL tests. The average maximum headcut depth and width at the conclusion of the experiments were also greater in the sand embankments than the clay embankments, approximately 180-200%.

Further inspection of the results outlined in Figure 2 shows that there was considerable variability between tests for a given soil class. For the sand tests, Test 7 was an outlier relative to Tests 5 and 9, which were compacted on the dry side of optimum, a condition shown to have a steeper erosion gradient and, consequently, greater erosion susceptibility (Hanson and Hunt, 2007). Three approximately equal-sized channels were observed to form during CL Test 2, whereas the other two CL tests resulted in less overall channel depth erosion. CL Test 10 was also unique in that it experienced a catastrophic slump over a large section of the landward face; any correlation between slope and slumping is uncertain. Interestingly, the embankment slope appears to have had no significant impact on headcut evolution in either soil case. Inflow rate, packing density, and initial water content (Table 2), relative to the OWC and MDD (Table 1), played a more important role in explaining the variability between results. The reader is referred to Section 3.3 for further discussion regarding inter- and intra-soil variability.

3.2. Time Scaling

Subsequent analysis of the results (i.e., erosion rate and mass loss) has a temporal component that must be considered. Time scaling, however, remains an open-ended question within the centrifuge

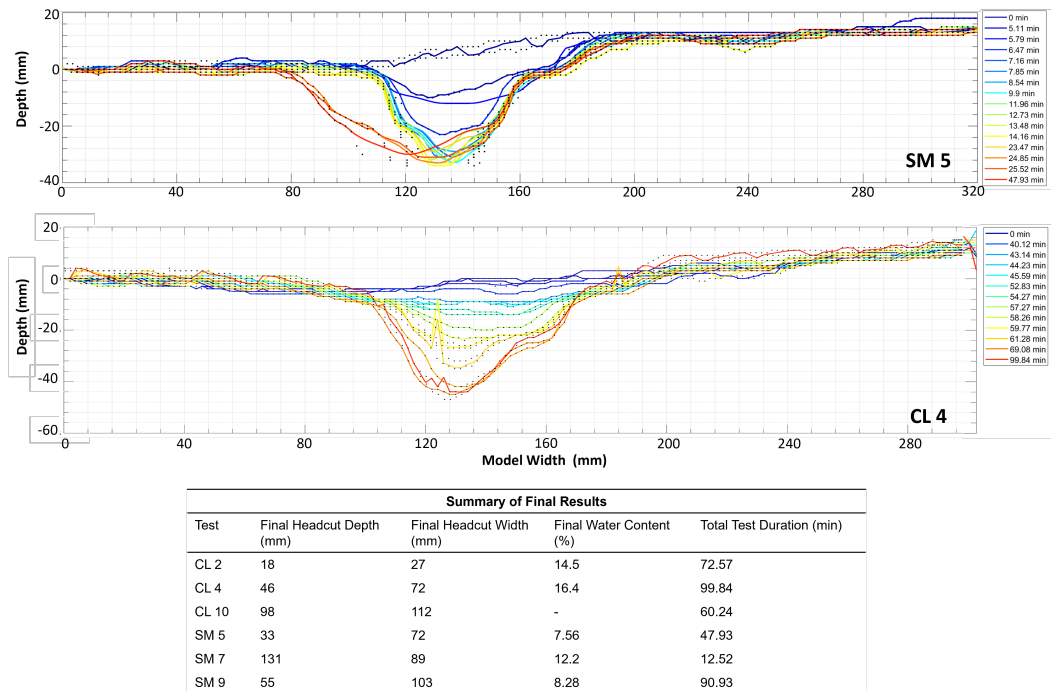


Figure 2. Centrifuge test cross-sections for SM Test 5 and CL Test 4 in model scale. The black dots in the images are the real data points measured by the RealSense camera, and the colored lines are the fitted lines.

modeling community, particularly in the context of erosion. This is best emphasized by the fact that mass was lost from the embankment during the centrifuge experiments of this study via two primary mechanisms: progressive surface erosion (i.e., channeling) and sudden sloughing events, which have different associated time scales. As previously noted, one of the primary objectives of this study is to determine the most appropriate time-scaling factor(s) for this context.

Goodings (1984) provides a theoretical foundation, based on similarity scaling, on which to base this analysis. Goodings (1984) explored scaling in terms of flow, initiation of erosion, and fully established sediment transport ultimately concluding that a scaling factor of N was appropriate for overflow and the initiation of erosion whereas a scaling factor $N^{1/2}$ satisfied sediment transport similarity requirements irrespective of the soil being cohesive or non-cohesive in nature. One can therefore posit that a N scaling factor would be applicable to Stage I and early Stage II overtopping; $N^{1/2}$ would likely be appropriate during Stage III and IV after the embankment is breached. It should be noted that similarity requires that the permeability and diameter of the model soil be reduced by a factor of N for overflow and erosion/transport; strict adherence to this was not possible in this study given the desire to use soils with the same soil classification and similar compaction characteristics as Hanson et al. (2005). Centrifuge studies have also applied a scaling factor of N and $N^{1/2}$ to capture the rapidity of events like

landslides and other granular/debris flows (Bowman et al., 2010; Turnbull et al., 2015; Cabrera et al., 2016; Song et al., 2017; Gue et al., 2010; Ng et al., 2023) which may be appropriate for the observed slumping in this study; Cabrera et al. (2020) interestingly even suggested an empirical $N^{2/3}$ scaling factor for dry granular flows. Note that ensuring exact similitude between the model and prototype scales requires that other system variables be scaled in addition to the geometry: particle size by $N^{1/2}$, permeability by N , or viscosity by N . None of these scaling considerations were again applied in the centrifuge experiments conducted for this, for the reasons above and concerns over potential damage to the pump if a high-viscosity fluid was used. A scaling factor of N^2 , while highly unlikely, is also considered for completeness; this scaling factor is commonly applied in seepage flow, diffusion, and consolidation problems (Taylor, 2018).

The above scaling considerations, and the process of soil erosion in general, are informed by three primary dimensionless numbers: the Reynolds number, the Froude number, and Shields number. The reader is referred to Goodings (1984) for an in-depth derivation of the scaling factors from these dimensionless numbers. Subsequent discussion of the dimensionless numbers is constrained to how they provide insight into observed erosion behavior. The Reynolds number, Re , is defined as the ratio of the inertial forces and the viscous forces in a fluid and is used to determine whether the flow regime is laminar or turbulent; the former have more kinetic energy

than the latter, creating conditions more conducive for erosion. The magnitude of the estimated values of Re presented in Table 3 exceeds a value of 2,300 commonly identified as the transition point from laminar to turbulent flow. The estimated Froude number (Fr), or the ratio between the inertial forces and gravitational forces, exceeds unity across all six tests (Table 3); this is indicative that flow conditions may be supercritical and that shear forces acting on the soil surface are sufficiently high to initiate and support erosion. Finally, the Shields number (Sh) can be used to determine whether conditions are favorable for initiating erosion from the start. It is defined as the ratio of the fluid forces acting on a soil particle to its overall weight (often expressed in terms of the d_{50}). The values provided in Table 3 are in excess of the critical Shields number (Sh_c), which is calculated using critical shear stress values (Julien, 2002), meaning erosion would be expected as observed in the centrifuge tests.

Table 3. Experimental Dimensionless Numbers

Test Number	Re	Fr	Sh	Sh_c
CL Test 2	2.57×10^6	5.01	2.99	0.04
CL Test 4	0.46×10^6	1.64	2.86	0.04
CL Test 10	0.42×10^6	2.31	2.63	0.04
SM Test 5	1.71×10^6	1.60	2.24	0.67
SM Test 7	1.68×10^6	1.15	1.34	0.67
SM Test 9	0.18×10^6	1.54	0.86	0.67

Given the high fidelity of the Hanson et al. (2005) datasets, the associated temporal scaling of the centrifuge experimental results is explored herein through a direct comparison of headcut channel widening. Figures 3 and 4 compare the temporal evolution of headcut widening as measured by Hanson et al. (2005) and the centrifuge experiments for the CL and SM soils, respectively. Note that the width of the channel is normalized to the maximum observed value in each test. This helps address variability among individual tests for a given soil type observed in both the 1G and centrifuge experimental series. The normalization facilitates a direct comparison of the curve shape and magnitude. The headcut width was used in this analysis in lieu of other temporal available data common to both studies such as the headcut progression which is more arbitrarily defined and difficult to quantify.

The three possible scaling factors ($N^{1/2}$, N , N^2) discussed above were independently applied to the temporal cross-section widths discussed in Section 3.1; lines were fit to the scaled centrifuge data to improve readability and not statistical analysis. The results can be approached from either the perspective of curve shape (i.e., rate of widening) or the overall magnitude (i.e., total duration of Stage I and II overtopping). Given the variability and unpredictability of the progression of erosion and the suddenness of slumping/sloughing events which are driven in part by soil type and compaction characteristics (density and water content), we have opted to focus

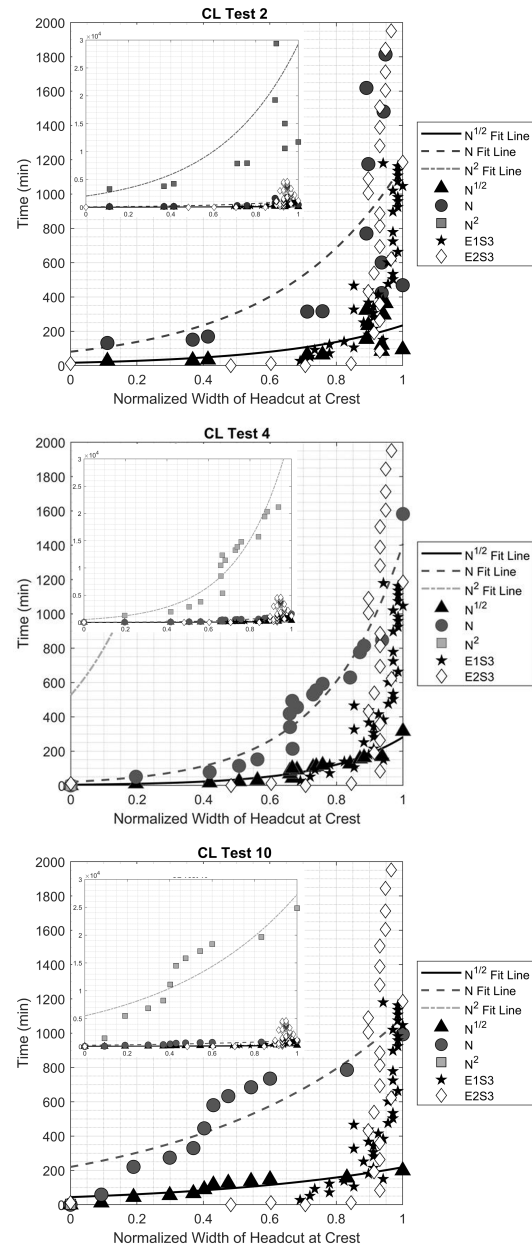


Figure 3. Comparison of centrifuge time scaling to CL Test E1S3 and E2S3 of Hanson et al. (2005).

on capturing the end-state, or time that the maximum headcut width occurs during Stage I and II overtopping given it ultimately informs the extent of the structural damage incurred by an embankment.

Hanson et al. (2005) observed that the headcut reached its maximum width between 1000 and 1200 minutes in their two CL soil tests (E1S3 and E2S3). The comparison of the 1G and centrifuge CL test results in Figure 3 immediately shows that a

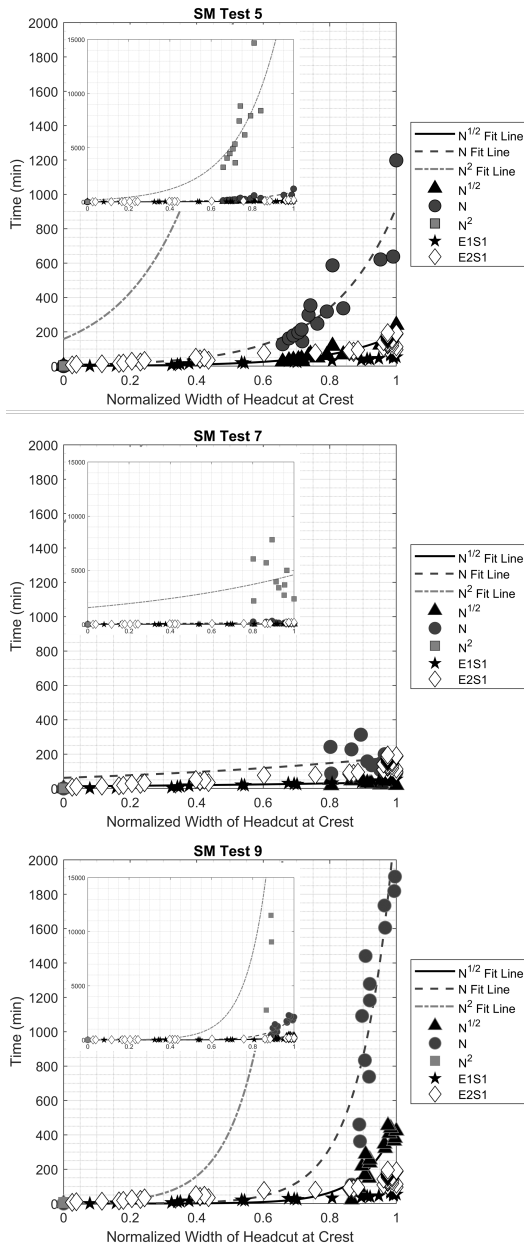


Figure 4. Comparison of centrifuge time scaling to SM Test E1S1 and E2S1 of [Hanson et al. \(2005\)](#).

scaling factor of N^2 yields headcut widening that is too slow. Using this factor would result in an overtopping event that unrealistically lasts 40 days, roughly 30 times longer than the [Hanson et al. \(2005\)](#) experiments. These results further suggest unrealistically that no significant erosive loss of embankment material would occur during the first 2.5 days of an overtopping event. As previously discussed, the poor performance of this scaling factor is to be expected given that erosion is a dynamic phenomenon; N^2 is better reserved for

slow processes such as seepage. While a scaling factor of N and $N^{1/2}$ both provide better fits to the 1G data, they still arguably fail to adequately capture the overall behavior across all three tests. The $N^{1/2}$ scaling factor captures the rapid growth of the headcut from the start of the tests, but the total erosion process is ultimately over-accelerated. This results in the maximum headcut width being consistently reached in 200 to 300 minutes across CL Tests 2, 4, and 10 – 250 to 600% faster than observed by [Hanson et al. \(2005\)](#). The use of a scaling factor of N slows the overall process of headcut widening compared to $N^{1/2}$. It delays the initial formation of the headcut channel however, but does allow the maximum headcut width to be reached in a time period much closer to that of the 1G tests. The N scaling factor furthermore captures the overall curvilinear trend/shape with respect to how it asymptotically approaches the maximum channel width toward the end of Stage II overtopping.

Figure 4 provides a comparison of the centrifuge SM soil test results against those reported for the same soil (Test E1S1 and E2S1) in [Hanson et al. \(2005\)](#). Much like the CL tests, analysis of the curves immediately shows how a scaling factor of N^2 overpredicts (by more than 20 days) how long the headcut widening process takes. While the scaling factor of N is better than N^2 , it still fails to adequately capture the headcut widening behavior for the SM tests – it is still too slow (≈ 600 minutes), particularly in the case of SM Test 5. Note that SM Test 7 is the notable exception to this observation, given that the fitted line is biased by the lack of measurements resulting from the embankment's early failure. The 1G and centrifuge results presented in Figure 4 correlate the best when a $N^{1/2}$ scaling factor is applied. The better performance of this scaling factor confirms that erosion, particularly with respect to SM embankments in the context of overtopping, is a dynamic process; the SM soil embankments were subject to rapid continuous erosion punctuated by sudden sloughing of the headcut channel side walls.

Collectively, this analysis (Figures 3 and 4) shows that none of the three proposed scaling factors perfectly captures the observed end state of the prototype scale headcut widening results of [Hanson et al. \(2005\)](#) during Stage I and Stage II overtopping. Comparison of the centrifuge and full-scale experiments suggests that a logarithmic relationship between time and headcut widening would be appropriate. Much like the empirical $N^{2/3}$ scaling factor proposed by [Cabrera et al. \(2020\)](#), this has no theoretical centrifuge modeling basis however ([Taylor, 2018](#)). Within the constraints of the scaling factors considered in this study, the validity of using N^2 can be immediately ruled out as it leads to headcut widening over unrealistically long periods of time, irrespective of the soil type. A scaling factor of N appears to be most suitable for the cohesive CL soil whereas $N^{1/2}$ is consistently the most appropriate scaling factor for use in modeling the SM soils. These findings conflict with those of [Goodings \(1984\)](#) who argued that a scaling factor of N corresponds to the initiation of erosion regardless of soil type.

The $N^{1/2}$ scaling factor's better fit for the SM soils suggests that another factor or process is at play. The SM soils were subject to sudden channel widening as the side walls would lose their cohesive strength and slough. This aligns more closely with reported results for landslide centrifuge experiments which ultimately relied on the use of both N and $N^{1/2}$ (Bowman et al., 2010; Gue et al., 2010). Note that all subsequent analysis in this study will utilize scaling factors of $N^{1/2}$ and N for the SM and CL soils, respectively. The authors acknowledge that these findings are not definitive, and that the selection of these scaling factors is based entirely on their overall performance in this study; further centrifuge and IG testing is needed to increase confidence in their use. It should be noted that there are centrifuge techniques that could be applied that could help overcome the scaling issue altogether. Increasing the fluid viscosity to reduce the time scales (Adamidis and Madabhushi, 2015), for example, could be used but would require the use of new hardware (i.e., pumps) in addition to the overall impacts on erosion being uncertain.

3.3. Erosion Rate and Total Volume Loss

The scaling factors selected in the previous section were used to calculate volumetric soil loss from the levee as a function of time (Figure 5). This helps provide a quantitative assessment of the damage sustained by the embankments as a direct result of overtopping. The volume loss was determined by integrating the D455 camera point clouds over time relative to a measurement made prior to the start of the experiments; these point clouds captured the formation and evolution of the primary headcut channel and any secondary rills/gullies in sufficient detail to facilitate volume calculations. In general, the SM tests results presented in Figure 5 showed significantly more volume loss than the CL tests which is consistent with the Hanson et al. (2005) datasets as well as historical overtopping (Flynn et al., 2021); the SM embankments lost 15-25 m³ more soil material than the CL embankments. The total volume losses for SM tests 5, 7, and 9 corresponded to a 6.5, 9.6, and 5.2% reduction in volume compared to the original embankment volume. Soil losses from CL Test 2, Test 4, and Test 10 similarly corresponded to a 3.2, 4.1, and 1.7% reduction in contrast. Further analysis of the results show that the majority of the soil loss occurred shortly after the initiation of the overtopping process in the case of the SM embankments. The CL soils typically experienced more gradual soil loss, occurring predominantly 250 to 800 minutes into the overtopping process.

Figure 6 presents the temporal evolution of the primary headcut channel's depth during the experiments. Results show that the SM soils typically displayed the deepest channelization when compared to the CL tests over the same period of time (i.e., 0-450 minutes). As in the case of channel widening, the headcut deepened the fastest in the case of the SM soils. The primary headcut depth time series data was used to estimate the erosion rate associated with Stage I and Stage II overtopping. A linear fit was applied to these measurements corresponding to the active widening of the headcut

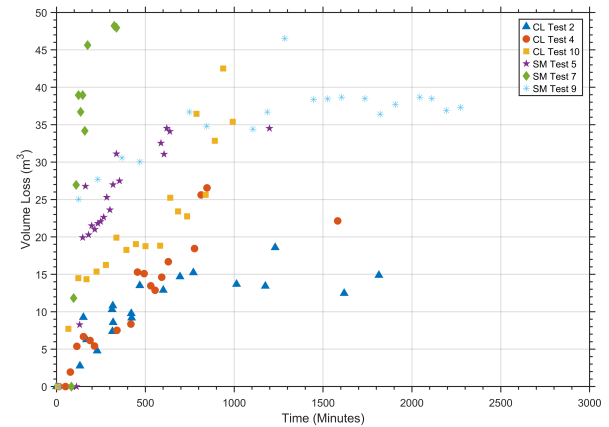


Figure 5. Estimated volume loss with time in levee section.

Table 4. Erosion rates and total volume loss

Test Number	Peak Velocity (m s ⁻¹)	Erosion Rate (mm hr ⁻¹)	r ²
CL Test 2	8.6	41.40	0.70
CL Test 4	2.3	84.17	0.87
CL Test 10	2.8	65.56	0.97
SM Test 5	3.5	1134.21	0.80
SM Test 7	2.8	755.8	0.93
SM Test 9	1.6	573.64	0.67

only; the average erosion rates are summarized in Table 4, along with their respective r² values and measured peak flow velocities. Results confirm that the SM soil eroded more quickly compared to the CL soil, by more than one to two orders of magnitude irrespective of slope. This can be attributed to a combination of different factors, including differences in the average flow velocity, which differed based on the geometry of the headcut channel (i.e., depth and width), as well as the general soil characteristics. Lower outflow velocities, for example, were often observed in the tests with the widest headcut channels as the flow was not concentrated to a small cross-sectional area. The CL soil embankments were furthermore less prone to erosion than their coarse-grained (i.e. SM) counterpart given CL having greater interparticle cohesion (Kelly and Gularte, 1981) and a lower permeability resulting in higher matric suction (Simon and Collison, 2001; Rahardjo et al., 2012) and overall optimum water content than the SM soil (Hunt et al., 2005).

The erosion rates and peak velocity data are plotted together in Figure 7 following Briaud (2008) to determine erosion classification. This chart provides insight into the erodibility, or overall susceptibility, of a soil to erosion. Results indicate that the SM soil can be classified as being highly erodible (II) to medium

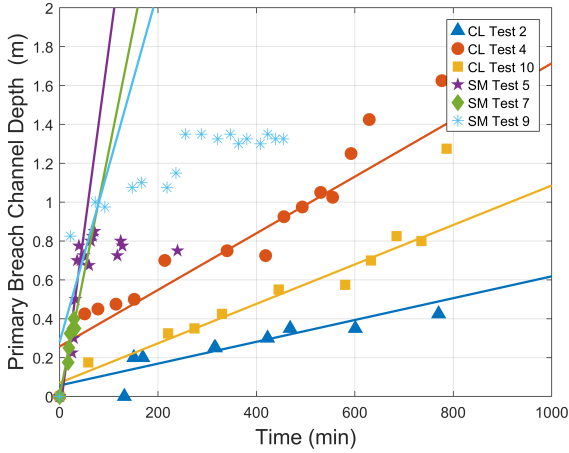


Figure 6. Headcut channel widening over time used to calculate erosion rate.

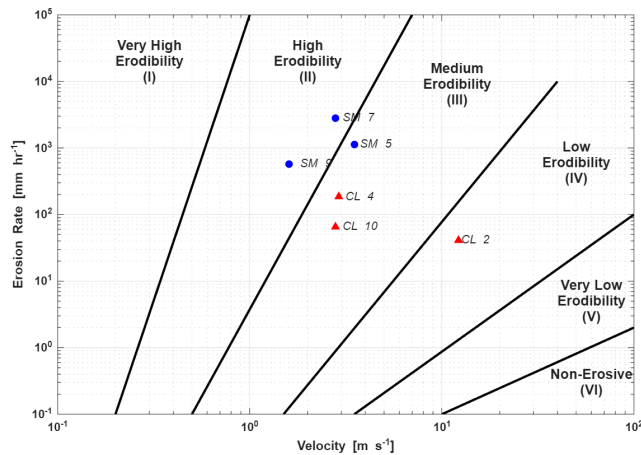


Figure 7. Erosion classification chart developed by Briaud (2008) with data from centrifuge tests indicated.

erodible (III) whereas the CL soil is classified as being low erodible (IV). These findings are in good agreement with Briaud et al. (2017) who compiled data on 84 different soil samples ranging from sands to clays and found that the former typically fall in Class II-III whereas the latter typically fall in Class III-IV.

The erodibility of an embankment is commonly discussed in terms of a stress-based detachment rate coefficient (k_d) which can be determined from the fundamental excess stress equation which, when modified for headcut widening, can be expressed as (Hunt et al., 2005):

$$(1) \quad \frac{dW}{dt} = 2k_d(\tau_e - \tau_c) \quad .$$

The factor of 2 included in Equation 1 is included to account for erosion from both sides of the headcut channel (Hunt et al., 2005). The effective stress on the sides of the channel can be estimated as 70% the effective stress of the bed of an open channel (Chow, 1959) which is given as (Temple et al., 2005):

$$(2) \quad \tau_e = \gamma h S \quad ,$$

where S , the energy grade line slope, can be calculated by rearranging Manning's equation:

$$(3) \quad S = \left(\frac{gn^2}{h^{1/3}} \right) \quad .$$

The flow depth normal to the surface of the levee surface can be estimated in terms of a critical depth which is 2/3 of the depth of the upstream reservoir. Combining Equations 2 and 3 and substituting the result into Equation 1 yields the final expression for the detachment rate coefficient (k_d):

$$(4) \quad k_d = \left(\frac{\frac{dW}{dt}}{2(\tau_e - \tau_c)} \right) \quad .$$

The critical stress is negligible compared to the sidewall effective stress and can therefore be excluded following the analysis of Hunt et al. (2005).

Figure 8 presents the measured temporal change in the headcut as a function of water content and compaction density. The results from the large scale USDA-ARS tests (Hanson et al., 2005) are included as well for validation of trends observed between the two different scales. Analysis shows that soils with lower water contents below are more susceptible to erosion which is in good agreement with USDA-ARS observations (Hunt et al., 2005; Hanson et al., 2005; Temple et al., 2005). The change in breach width with time in contrast, appears to be uncorrelated to the compaction density, also in good agreement. The USDA-ARS found that a 2% change in water content on the dry side of optimum could alter k_d by two orders of magnitude, while similar changes wet of optimum produced relatively minor effects. Compaction near optimum water content consistently produced the most erosion-resistant soil structure, with k_d values ranging from 0.01 to over 100 cm³N⁻¹s⁻¹ for the same soil under different compaction conditions. The calculated values of k_d are presented in Figure 9 against data compiled by Hanson et al. (2010). These latter data were derived from large-scale flume scour tests, headcut migration tests, and JET testing. Results show that the k_d values estimated from the centrifuge data for the CL and SM tests both fall within the range reported by Hanson et al. (2010). Overall, these results help to validate the chosen time scaling factors discussed in the preceding section. The higher k_d values for the SM soils indicate greater erodibility, again confirms the findings of Hanson and Temple (2002); Hunt et al. (2005) who observed that soils with lower water contents are more susceptible to soil erosion. These results furthermore, borrowing from Hanson et al. (2010), "provide evidence of the coherence of erodibility measurements at

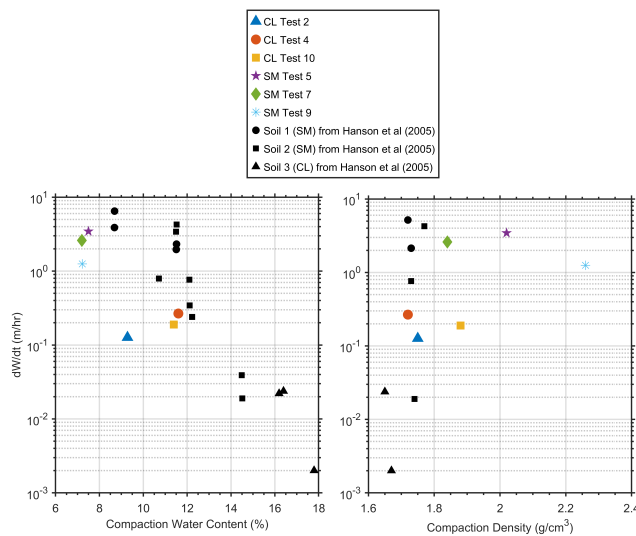


Figure 8. Change in breach width over time as determined from centrifuge testing and [Hunt et al. \(2005\)](#). Results are presented as a function of the levee's water content and compaction density.

the large and small scales" across multiple different measurement approaches.

4. Conclusions

The process of embankment overtopping is dependent on many factors including breach dimensions, time, soil type and conditions, reservoir size and inflow, and overflow depth. The importance of these variables and the underlying processes remains poorly understood, with understanding limited primarily to a few post-failure investigations and relatively small 1G experiments (field and flume). Centrifuge modeling offers a unique alternative to expensive (logistically and financially) 1G experimentation or dependency on real-life embankment failures.

The overall objective of this work was to conduct a series of controlled centrifuge experiments modeled after the 1G overtopping failure experiments performed by the USDA-ARS ([Hanson et al., 2005](#)). The goal was to generate datasets that were complementary and more expansive, incorporating cross-sections, erosion rate, and total volume loss analysis. Results of the centrifuge experiments, breach channel width evolution, were generally in good agreement with those of [Hanson et al. \(2005\)](#). The embankments constructed from SM soils were more susceptible to erosion and resulted in greater volumetric soil loss than those constructed from CL soils. The headcut channel geometry and rate of erosion was also significantly more variable for the SM tests. Compaction effort (i.e., bulk density) and the slope of the levee were furthermore shown to play an important role in determining

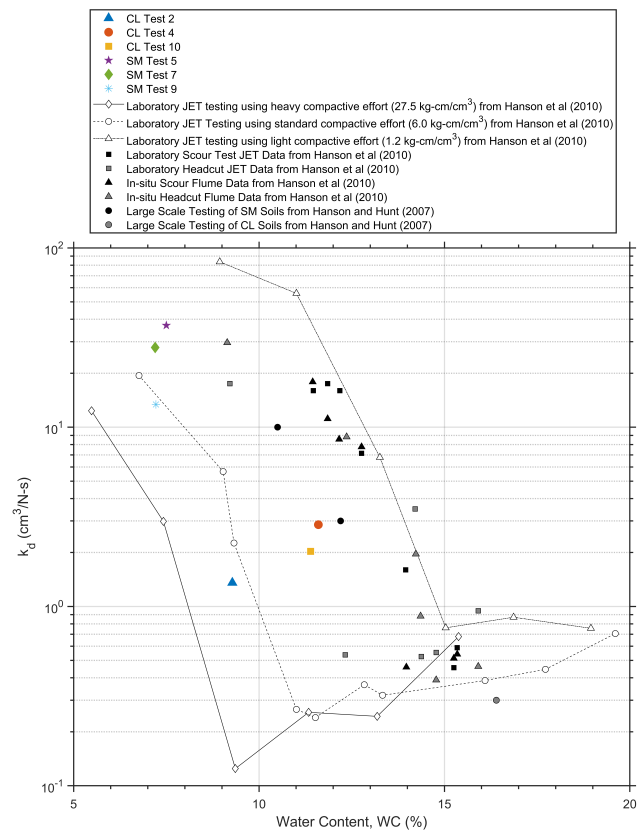


Figure 9. Comparison of detachment rate coefficient values calculated from centrifuge experimental data against values reported in [Hanson et al. \(2010\)](#).

erosion rates and total volume loss. Comparison of the results against the classification scheme of [Briaud \(2008\)](#) showed that the SM soils were of a Class II-III (medium to high erodibility) whereas the CL soils were of Class III and IV (low erodibility)

The dataset of [Hanson et al. \(2005\)](#) was furthermore extremely valuable for centrifuge modeling, as it provided a basis for comparison necessary to explore appropriate time-scaling factors, which remain highly uncertain in the context of erosion processes. It could be inferred from the analysis of the centrifuge experimental results that a scaling factor of N was the most appropriate for the CL soils and $N^{1/2}$ for the SM soils. These scaling factors correlated well with the values of [Hanson et al. \(2005\)](#) for both of these soil types. When these factors were used to calculate the value of the detachment rate coefficient, k_d , the resulting values fell within a wide range of values presented in [Hanson et al. \(2010\)](#) for SM and CL soils measured at different water contents using different measurement techniques (laboratory JET, flume, full-scale overtopping). The correlation of the centrifuge results with these reported values lends credence to the validity of the chosen time

scales applied in the present study and also confirms a coherence of scale with respect to techniques used to measure k_d .

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